Helicon Waves in Solids with a Closed Fermi Surface*

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We present a theory of helicon wave propagation in solids with a closed Fermi surface of arbitrary shape. In the limit of a high magnetic field, the dispersion and polarization are independent of band structure. The electric field of the wave is elliptically polarized in a plane almost normal to the static magnetic field $\mathbf{H}_0$, and the magnetic field of the wave is circularly polarized in a plane normal to the wave vector $\mathbf{q}$. On the other hand, the damping of the wave is *qualitatively* changed by anisotropy of the Fermi surface. We find that the Landau damping may exhibit several minima for certain special directions of $\mathbf{H}_0$ with respect to $\mathbf{q}$ and the crystalline axes. On the other hand, the Landau damping does not vanish, in general, even for $\mathbf{H}_0 \| \mathbf{q}$. This behavior is explained by the presence of cyclotron orbits which, in real space, are not perpendicular to $\mathbf{H}_0$. Although the theory does not restrict the value of $q\ell$ (where ℓ is the mean free path), we also derive the behavior in the limit $q\ell \ll 1$. This case may be of interest for an experimental investigation of mean free paths in metals. As a particularly instructive example, we also study the special case of an ellipsoidal Fermi surface.

I. INTRODUCTION AND BASIC ASSUMPTIONS

IN the presence of a high magnetic field, electromagnetic waves of frequency below the plasma frequency may propagate in solids with small attenuation. For propagation along the magnetic field ($\mathbf{H}_0 \| \mathbf{q}$, where $\mathbf{H}_0$ is the static magnetic field and $\mathbf{q}$ is the wave vector) the waves considered are circularly polarized. They are called "helicons"[1] because of this property. Ignoring damping, it is easy to show that the drift velocity is perpendicular to the electric field of the wave, which is the reason for the stability of this excitation. In fact, there are three mechanisms which may lead to damping of the wave.

Doppler-shifted cyclotron resonance. An orbiting electron which moves along the magnetic field with an average velocity $\mathbf{v}_0$ (see Fig. 1) experiences the wave at a frequency $\omega_{\text{eff}} = \omega - \mathbf{q} \cdot \mathbf{v}_0$. When $\omega_{\text{eff}} = \omega_c$ (where ω_c is the cyclotron frequency) for any cyclotron orbit on the Fermi surface, strong resonant interaction between these electrons and the wave inhibits its propagation. There will be no group of electrons satisfying this condition provided that $\omega + q_z(v_0)_{\max} < \omega_c$. We shall assume that ω is far away from the absorption "edge," i.e., that $\omega/\omega_c \ll 1$ and $q_z(v_0)_{\max}/\omega_c \sim qR \ll 1$, where R is of the order of the largest cyclotron radius that electrons on the Fermi surface may have. These inequalities indicate that an electron experiences an *almost* constant wave field.

Collisional damping. Because of various scattering mechanisms, the electrons have a finite collision frequency ν. An electron should be able to perform many orbits before it is scattered, i.e., we require $\omega_c/\nu \gg 1$. We then have a small but finite damping which arises from a deviation $\sim \nu/\omega_c$ from the right angle between the drift velocity and the electric field. The current is at all times perpendicular to the wave vector.

If open orbits exist for a certain direction of the magnetic field, the effective cyclotron frequency is zero and the damping becomes extremely large. Other propagation characteristics also exhibit extreme anisotropic behavior. The influence of open orbits was studied both theoretically[2] and experimentally.[3] In this work, we shall assume that there are no open orbits.

An additional assumption is that the collision frequency ν does not depend on the phase of an electron in its cyclotron orbit. Thereby we achieve considerable simplification, while formally we take into account possible effects of anisotropy of ν. A special conference[4] has been recently devoted to such effects.

Collisionless (Landau) damping. This type of damping was originally predicted by Landau[5] for longitudinal oscillations in a collisionless ($\nu \to 0$) plasma. A simple physical picture of the damping was given by Bohm and Gross.[6] The existence of collisionless damping of helicon waves was pointed out, and the magnitude of the damping was calculated by Kaner and Skobov.[7] In fact, the mechanisms are quite different, and in our case it is the magnetic field of the helicon which is responsible for the "Landau" damping (of course, ultimately the power is delivered to the electrons by means of the induced *electric* field). The mechanism of this "magnetic"

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[1] For reviews on helicon waves see (a) *Proceedings of the Seventh International Conference on the Physics of Semiconductors, Paris, 1964* (Academic Press Inc., New York, 1965); (b) D. P. Morgan, Phys. Status Solidi **24**, 9 (1967); (c) J. Mertsching, *ibid.* **14**, 3 (1966); **26**, 9 (1968); (d) Ref. 13.

[2] C. Kittel, *Quantum Theory of Solids* (John Wiley & Sons, Inc., New York, 1963), p. 322; S. J. Buchsbaum and P. A. Wolff, Phys. Rev. Letters **15**, 406 (1965).

[3] C. C. Grimes, G. Adams, and P. H. Schmidt, Phys. Rev. Letters **15**, 409 (1965); J. R. Merrill, Phys. Rev. **166**, 716 (1968).

[4] *Proceedings of an International Conference on Electron Mean Free Paths in Metals, Zurich, 1968*, edited by R. G. Chambers, J. L. Olsen, and J. M. Ziman (Springer-Verlag, Berlin, 1968).

[5] L. D. Landau, J. Phys. USSR **10**, 25 (1946).

[6] D. Bohm and E. D. Gross, Phys. Rev. **75**, 1851 (1949); **75**, 1864 (1949).

[7] E. A. Kaner and V. G. Skobov, Zh. Eksperim. i Teor. Fiz. **45**, 610 (1963) [English transl.: Soviet Phys.—JETP **18**, 419 (1964)].

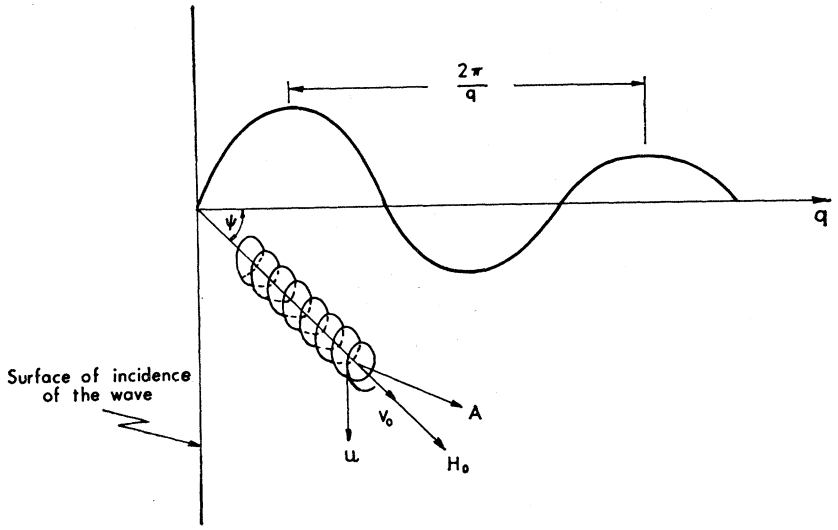

FIG. 1. Dynamics of an electron in an anisotropic medium: In addition to a constant mean velocity $\mathbf{v}_0$ along $\mathbf{H}_0$, the electron has an orbital velocity $\mathbf{u}$; the plane of the orbit is normal to a vector $\mathbf{A}$, which, in general, is not parallel to $\mathbf{H}_0$. The diagram also shows that the wave propagates at right angles to the surface of incidence and has a wavelength much larger than the cyclotron radius ($qR \ll 1$).

Landau damping was explained by Buchsbaum and Platzman[8] along the following lines.

When the magnetic field is tipped with respect to the wave vector, the magnetic field of the helicon (being polarized in a plane perpendicular to $\mathbf{q}$) has an oscillating component along it. Thus, the lines of force will alternatively diverge and converge. The radial component of the field will exert a Lorentz force on the orbiting electron; it will be speeded up where the lines of field diverge and slowed down where they converge. If the mean path $l(\sim v_F/\nu)$ of the electron is much larger than a wavelength ($ql \gg 1$), the energy interchange will average out to zero for *almost* all electrons. The exceptions are those which have a component of velocity along the wave vector (averaged over a cyclotron orbit) very nearly equal to the wave velocity ($\mathbf{v}_0 \cdot \mathbf{q} \cong \omega$). These will be caught in the potential troughs of the wave and carried along with it, giving rise to resonant interaction. Those which have $\mathbf{v}_0 \cdot \mathbf{q}$ infinitesimally smaller than ω will be accelerated on the average, while those which have $\mathbf{v}_0 \cdot \mathbf{q}$ infinitesimally bigger than ω will be decelerated on the average. Since in thermal equilibrium there are more slow than fast particles ($\partial f_0/\partial v_0 < 0$), there will be a net extraction of energy from the wave by the electrons.

In this discussion, collisions were implicitly assumed only through the requirement that the system be very close to a state of thermal equilibrium. In fact, although Landau damping exists even in the limit $\nu \to 0$, it is modified by a finite collision frequency. Collisions tend to diminish the number of electrons which move coherently with the wave.[9] On the other hand, electrons for which $\mathbf{v}_0 \cdot \mathbf{q}$ differs significantly from ω will now also be able to contribute to the Landau damping, for those which traverse in a collision time $1/\nu$, a distance smaller than a wavelength will, on the average, gain energy.

[8] S. J. Buchsbaum and P. M. Platzman, Phys. Rev. **154**, 395 (1967).

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The collision-modified Landau damping was calculated by Buchsbaum and Platzman[8] for an isotropic Fermi surface and measured by Houck and Bowers.[10] In the alkali metals, the damping is a continuously increasing function of the "tipping" angle $(\mathbf{H}_0, \mathbf{q})$.

In the present work, we wish to investigate how the anisotropy of the Fermi surface affects helicon propagation. In Sec. III, we shall see that in the high-field limit it does not affect the dispersion and polarization of the wave. (If our requirement $qR \ll 1$ does not hold, this will no longer be true.[11,12])

In Sec. IV, we analyze the damping of the wave. Although the value of $q_z l$ is not restricted, we pay special attention to the cases $q_z l \sim 1$ and $q_z l \ll 1$ (local limit), which have not yet been investigated. The case $q_z l \gg 1$ (extreme nonlocal limit) has been investigated by Kaner and Skobov[13,14] and by McWhorter and Walpole.[15] In this connection, it may be noted that McWhorter and Walpole's[15] assertion that only $\omega_c/\nu \gg 1$ is required in their theory is based on an approximation[16] which is valid only in the case $q_z l \gg 1$. Thus, their derivation does not hold for an arbitrary value of $q_z l$. In

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[11] J. L. Stanford and E. A. Stern, Phys. Rev. **144**, 534 (1966); P. R. Antoniewicz, Phys. Letters **24A**, 83 (1967); N. B. Brovtsyna and V. G. Skobov, Zh. Eksperim. i Teor. Fiz. **56**, 694 (1969) [English transl.: Soviet Phys.—JETP **29**, 379 (1969)].

[12] J. N. Walpole and A. L. McWhorter, Phys. Rev. **158**, 708 (1967); A. L. McWhorter and J. N. Walpole, *ibid.* **163**, 618 (1967).

[13] E. A. Kaner and V. G. Skobov, Advan. Phys. **17**, 605 (1968). Note that some of the earlier results of Kaner and Skobov (Ref. 14) have been corrected in the present reference.

[14] E. A. Kaner and V. G. Skobov, Zh. Eksperim. i Teor. Fiz. **46**, 1106 (1964) [English transl.: Soviet Phys.—JETP **19**, 749 (1964)]; Usp. Fiz. Nauk **89**, 367 (1966) [English transl.: Soviet Phys.—Usp. **9**, 480 (1967)]. See note in Ref. 13.

[15] A. L. McWhorter and J. N. Walpole, Phys. Rev. **158**, 719 (1967).

[16] In Ref. 15, Eq. (2.18), the authors neglect a factor of the order of $\exp(-i\mathbf{q} \cdot \mathbf{v}_0/\omega_c)$ in the integrand. However, if the condition $q_z l \gg 1$ does *not* hold, noncentral orbits also take part in the damping and $v_0 \sim v_F$. Thus, for noncentral orbits $v_0/\omega_c \sim R$. The neglected factor is then of the same order as another factor in the integrand, $\exp(-i\mathbf{q} \cdot \mathbf{R})$, which plays an important role.

the special case $q_z l \gg 1$ their result agrees with that of Ref. 13 (except for a numerical factor).

Quinn[17] and McWhorter and Walpole[15] pointed out that even for propagation along the field ($\mathbf{H}_0 \| \mathbf{q}$), in general, the Landau damping does not vanish. This was explained[15] by tilted cyclotron orbits in real space, caused by anisotropy of the Fermi surface. The effect was observed by Walpole and McWhorter[12] in PbTe.

Another particularly interesting feature of the damping, and a direct manifestation of anisotropy, are minima, obtained for special directions of the magnetic field with respect to the wave vector and the crystalline axes. Such minima were found in indium[18] by Krylov[19] and by Halevi, Lipson, and Rabinovitch.[20] This behavior is in sharp contrast with that of the alkali metals.[10] Thus, it may be said that the damping in a solid with an anisotropic Fermi surface differs *qualitatively* from that in a solid with an isotropic Fermi surface. This effect was briefly discussed by Halevi[21] in a preliminary note. We shall endeavor to give a thorough microscopic interpretation in Sec. IV.

An additional assumption is that the temperature is low enough for the Fermi distribution to be considered degenerate, while high enough for excluding any quantum effects ($\hbar\omega_c \ll kT \ll \epsilon_F$). Nevertheless, in Sec. IV, we shall comment briefly on the connection between quantum oscillations in the attenuation of helicon waves and anisotropy of the Fermi surface.

We shall conclude this paper (Sec. V) with an analysis of the simplest model which exhibits anisotropy, an ellipsoidal Fermi surface. In fact, it was this model which led us to an interpretation of the minima in indium.[20,21] In addition, since the Fermi surfaces of semimetals and semiconductors are composed, to a good approximation, of variously oriented ellipsoids, there is a practical value as well. Although the ellipsoidal Fermi surface has been treated by several authors[12,17,22] in connection with helicons, some interesting points remain to be discussed.

II. CONDUCTIVITY TENSOR

The properties of the medium are described by the conductivity tensor of the charge carriers. It depends on frequency and wave number, as well as on the static magnetic field. The elements of the conductivity tensor may be found either by a formal solution of the transport equation,[7,13,23] or by a kinetic approach due to Chambers.[13,24] For a Fermi surface of arbitrary shape, i.e., a general energy-quasimomentum relationship $\epsilon(\mathbf{p})$, they are given by the expression[25]

$$\sigma_{ij} = \frac{2ec}{h^3 H_0} \sum \int_{-p_{zm}}^{p_{zm}} dp_z \frac{m_c^2}{1-\exp[-2\pi(\nu - i\omega + i\mathbf{q}\cdot\mathbf{v}_0)/\omega_c]} \times \int_0^{2\pi} d\tau\, v_i(\tau) \int_{\tau-2\pi}^{\tau} d\tau'\, v_j(\tau') \times \exp\left(-\int_{\tau'}^{\tau} d\tau'' \frac{\nu - i\omega + i\mathbf{q}\cdot\mathbf{v}(\tau'')}{\omega_c}\right). \quad (1)$$

The variable p_z labels a cyclotron orbit in phase space; it is the z component of the quasimomentum. (The z axis is chosen along $\mathbf{H}_0$.) Its maximal value p_{zm} is determined by the distance from the origin of the tangent plane to the Fermi surface which is perpendicular to $\mathbf{H}_0$. The cyclotron mass m_c is given by

$$m_c(p_z) = \frac{1}{2\pi}\left(\frac{\partial S}{\partial \epsilon}\right)_{p_z}, \quad (2)$$

where $S(\epsilon, p_z)$ is the area of a cross section of the Fermi surface, and

$$\omega_c(p_z) = eH_0/m_c c \quad (3)$$

is the cyclotron frequency. The phase variable $\tau = \omega_c t$ measures the time of flight of an electron along a cyclotron orbit. The instantaneous group velocity components of an electron

$$v_i(p_z, \tau) = \frac{\partial \epsilon}{\partial p_i} \quad (4)$$

are to be found explicitly from its equation of motion in a constant magnetic field (the wave fields are completely negligible). This is conveniently expressed in the following form:

$$\frac{dp_x}{d\tau} = -m_c v_y, \quad (5a)$$

$$\frac{dp_y}{d\tau} = m_c v_x. \quad (5b)$$

Averaging these equations over a cyclotron period we see that $\langle v_x \rangle = \langle v_y \rangle = 0$. Therefore, the electron moves in the average along $\mathbf{H}_0$. The magnitude of its average velocity is

$$v_0(p_z) = \frac{1}{2\pi} \oint d\tau\, v_z(p_z, \tau). \quad (6)$$

In Eq. (1), the collision frequency ν is assumed to be independent of τ, but not necessarily of p_z. Finally, integration is performed over all "slices" of the Fermi

[17] J. J. Quinn, Phys. Rev. **135**, A181 (1964).
[18] It is known that indium has a closed Fermi surface.
[19] J. P. Krylov, Zh. Eksperim. i Teor. Fiz. **54**, 1738 (1968) [English transl.: Soviet Phys.—JETP **27**, 934 (1968)].
[20] P. Halevi, S. G. Lipson, and K. Rabinovitch, Low Temp. Phys. **1**, 189 (1969).
[21] P. Halevi, Phys. Letters **27A**, 647 (1968).
[22] G. Simon, Solid State Commun. **2**, 255 (1964); P. R. Wallace, Can. J. Phys. **43**, 2162 (1965).
[23] M. Ya. Azbel and E. A. Kaner, Zh. Eksperim. i Teor. Fiz. **32**, 896 (1957) [English transl.: Soviet Phys.—JETP **5**, 730 (1957)].
[24] R. G. Chambers, Proc. Phys. Soc. (London) **A65**, 458 (1952).

[25] Reference 13, Eq. (89), with the substitutions (171) and $df_0/d\epsilon = -\delta(\epsilon - \epsilon_F)$.

surface with different values of p_z, and the summation is over all the carrier groups (i.e., sheets of the Fermi surface in different Brillouin zones).

We now require the conditions

$$\nu/\omega_c \ll 1, \tag{7a}$$

$$\omega/\omega_c \ll 1, \tag{7b}$$

$$q_z v_{0\max}/\omega_c \sim qR \ll 1. \tag{7c}$$

Since the limits of all integrals in Eq. (1) are finite, we may use the expansion

$$\sigma_{ij}=\frac{e^2}{\pi h^3}\sum\int_{-p_{zm}}^{p_{zm}} dp_z\, m_c \frac{1+\pi(\nu-i\omega+iq_zv_0)/\omega_c-\cdots}{\nu-i\omega+iq_zv_0}$$
$$\times\int_0^{2\pi} d\tau\, v_i(\tau)\int_{\tau-2\pi}^{\tau} d\tau'\, v_j(\tau')$$
$$\times\Big[1-\frac{1}{\omega_c}\int_{\tau'}^{\tau} d\tau''[\nu-i\omega+i\mathbf{q}\cdot\mathbf{v}(\tau'')]$$
$$+\frac{1}{2\omega_c^2}\Big(\int_{\tau'}^{\tau} d\tau''[\nu-i\omega+i\mathbf{q}\cdot\mathbf{v}(\tau'')]\Big)^2-\cdots\Big]. \tag{8}$$

We shall evaluate each element σ_{ij} in the lowest nonvanishing order in $1/H_0$ for a closed Fermi surface. The first term in the square brackets contributes only to σ_{zz}. All other elements in this order are proportional to either v_x or v_y integrated over a cyclotron period, and therefore vanish. The second term in the square brackets determines $\sigma_{ij}(i\neq j)$ in the lowest order, while we have to go to the third term to get a nonvanishing contribution to σ_{xx} and σ_{yy}. As to the expansion in the first integral, it is now clear that, in the lowest nonvanishing order in $1/H_0$, only the first term has to be kept. We now use Eqs. (5a) and (5b) for v_y and v_x and we integrate by parts (one integration for each component v_x or v_y). We also make use of the central symmetry of the Fermi surface. It is convenient to choose coordinate axes such that the static magnetic field $\mathbf{H}_0$ is along the z axis, and the wave vector $\mathbf{q}$ lies in the y-z plane, with

$$q_x=0,\quad q_y=q\sin\psi,\quad q_z=q\cos\psi. \tag{9}$$

The angle $\psi=(\mathbf{H}_0,\mathbf{q})$ is the tipping angle of the magnetic field. The calculations are straightforward though lengthy. The final results are

$$\sigma_{xx}=\frac{4\pi c^2}{h^3H_0^2}\sum\int_{-p_{zm}}^{p_{zm}} dp_z\, m_c$$
$$\times\Big((\nu-i\omega)\langle P_y^2\rangle+\frac{(\mathbf{q}\cdot\langle\mathbf{v}P_y\rangle)^2}{\nu-i\omega+iq_zv_0}\Big), \tag{10a}$$

$$\sigma_{yy}=\frac{4\pi c^2}{h^3H_0^2}\sum\int_{-p_{zm}}^{p_{zm}} dp_z\, m_c$$
$$\times\Big((\nu-i\omega)\langle P_x^2\rangle+\frac{(\mathbf{q}\cdot\langle\mathbf{v}P_x\rangle)^2}{\nu-i\omega+iq_zv_0}\Big), \tag{10b}$$

$$\sigma_{zz}=\frac{4\pi e^2}{h^3}\sum\int_{-p_{zm}}^{p_{zm}} dp_z\frac{m_cv_0^2}{\nu-i\omega+iq_zv_0}, \tag{10c}$$

$$\sigma_{yx}=(n_e-n_h)ec/H_0, \tag{10d}$$

$$\sigma_{zx}=-\frac{(n_e-n_h)ec}{H_0}\tan\psi+\frac{4\pi ec}{h^3H_0q_z}$$
$$\times\sum\int_{-p_{zm}}^{p_{zm}} dp_z\,(\nu-i\omega)\frac{m_c\mathbf{q}\cdot\langle\mathbf{v}P_y\rangle}{\nu-i\omega+iq_zv_0}, \tag{10e}$$

$$\sigma_{zy}=-\frac{4\pi ec}{h^3H_0q_z}\sum\int_{-p_{zm}}^{p_{zm}} dp_z\,(\nu-i\omega)\frac{m_c\mathbf{q}\cdot\langle\mathbf{v}P_x\rangle}{\nu-i\omega+iq_zv_0}. \tag{10f}$$

The remaining three off-diagonal elements may be obtained with the help of Onsager's relation

$$\sigma_{ji}(\mathbf{H}_0)=\sigma_{ij}(-\mathbf{H}_0). \tag{11}$$

All the averages are taken over a cyclotron cycle. The abbreviation

$$P_i=p_i-\langle p_i\rangle \tag{12}$$

denotes the oscillating part of the transverse components of the quasimomentum. The density n_k of a group of charge carriers is proportional to the volume of the corresponding sheet of the Fermi surface

$$n_k=2h^{-3}\Big(\int_{-p_{zm}}^{p_{zm}} dp_z\oint p_x\,dp_y\Big)_k. \tag{13}$$

There is a special case of interest, namely, when both $\mathbf{q}$ and $\mathbf{H}_0$ lie in a mirror plane of the Fermi surface. We then have

$$\mathbf{q}\cdot\langle\mathbf{v}P_x\rangle=q_y\langle v_yP_x\rangle+q_z\langle v_zP_x\rangle=0. \tag{14}$$

The first term vanishes in general, as may be seen by multiplying Eq. (5a) by p_x and averaging. The second term vanishes as a result of mirror symmetry in the plane $p_x=0$. In this case σ_{zy} [Eq. (10f)] and the second term of σ_{yy} [Eq. (10b)] vanish.

In many experiments, $\mathbf{q}$ and $\mathbf{H}_0$ lie in a mirror plane and are parallel to each other. Using Eq. (14) and substituting $\psi=0$, we may considerably simplify Eqs. (10a), (10b), (10e), and (10f) as follows:

$$\sigma_{xx}=\frac{4\pi c^2}{h^3H_0^2}\sum\int_{-p_{zm}}^{p_{zm}} dp_z\, m_c$$
$$\times\Big((\nu-i\omega)\langle P_y^2\rangle+\frac{(q\langle v_zP_y\rangle)^2}{\nu-i\omega+iqv_0}\Big), \tag{15a}$$

$$\sigma_{yy}=\frac{4\pi c^2}{h^3H_0{}^2}\sum\int_{-p_{zm}}^{p_{zm}}dp_z\,m_c(\nu-i\omega)\langle P_x{}^2\rangle, \tag{15b}$$

$$\sigma_{zx}=\frac{4\pi ec}{h^3H_0}\sum\int_{-p_{zm}}^{p_{zm}}dp_z\,(\nu-i\omega)\frac{m_c\langle v_zP_y\rangle}{\nu-i\omega+iqv_0}, \tag{15e}$$

$$\sigma_{zy}=0. \tag{15f}$$

Although for the special case of a spherical Fermi surface $\langle v_zP_y\rangle$ vanishes, this expression does not necessarily vanish for a complex Fermi surface. The reason for this is that, in general, v_z is not constant along the orbit. In other words, the velocity has, in general, an oscillating component along the magnetic field. Only in the case of an ordinary metal (but not a semimetal), with $\mathbf{H}_0$ and $\mathbf{q}$ parallel to a rotation axis of the second order of higher does the expression $\langle v_zP_y\rangle$ vanish, even without v_z being constant along the orbit.

III. DISPERSION AND POLARIZATION

The physical properties of the helicon waves may be derived from Maxwell's equations

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}, \tag{16}$$

$$\nabla\times\mathbf{H}=(4\pi/c)\mathbf{J}. \tag{17}$$

We have neglected the displacement current $(\partial\mathbf{D}/\partial t)/4\pi$ in Eq. (17). This is legitimate for frequencies such that

$$\omega\ll\omega_P{}^2/\omega_cK, \tag{18}$$

where

$$\omega_P=(4\pi ne^2/m)^{1/2} \tag{19}$$

is the plasma frequency, m is the free-electron mass, and K is the static dielectric constant (see below). Inequality (18) is always satisfied in ordinary metals; however, in general, it does not hold for helicon propagation in semiconductors.

Assuming plane waves

$$\mathbf{E},\,\mathbf{H}\propto\exp[i(\mathbf{q}\cdot\mathbf{r}-\omega t)], \tag{20}$$

Eqs. (16) and (17) yield

$$\mathbf{J}=\frac{ic^2}{4\pi\omega}\mathbf{q}\times(\mathbf{q}\times\mathbf{E}). \tag{21}$$

Obviously, the current rotates with a frequency ω in a plane perpendicular to $\mathbf{q}$. It may be also expressed in another way, namely, that

$$\mathbf{J}=\boldsymbol{\sigma}\cdot\mathbf{E}, \tag{22}$$

where $\boldsymbol{\sigma}$ is the conductivity tensor. Equating the right-hand sides of Eqs. (21) and (22), we obtain the following set of homogeneous equations:

$$[\sigma_{xx}+(ic^2q^2/4\pi\omega)]E_x+\sigma_{xy}E_y+\sigma_{xz}E_z=0, \tag{23a}$$

$$\sigma_{yx}E_x+[\sigma_{yy}+(ic^2q_z{}^2/4\pi\omega)]E_y+[\sigma_{yz}-(ic^2q_yq_z/4\pi\omega)]E_z=0, \tag{23b}$$

$$\sigma_{zx}E_x+[\sigma_{zy}-(ic^2q_yq_z/4\pi\omega)]E_y+[\sigma_{zz}+(ic^2q_y{}^2/4\pi\omega)]E_z=0. \tag{23c}$$

The determinant of this equation must vanish and this condition yields the dispersion relation

$$c^2q^2/2\pi\omega=(\beta/\alpha)\{\pm[(4\alpha\gamma/\beta^2)-1]^{1/2}+i\}, \tag{24a}$$

where

$$\alpha=\sigma_{zz}\cos^2\psi+\sigma_{yy}\sin^2\psi+(\sigma_{yz}+\sigma_{zy})\sin\psi\cos\psi, \tag{24b}$$

$$\beta=\sigma_{zz}(\sigma_{xx}\cos^2\psi+\sigma_{yy})-(\sigma_{xy}\sin\psi+\sigma_{xz}\cos\psi)\times(\sigma_{yx}\sin\psi+\sigma_{zx}\cos\psi)-\sigma_{yz}\sigma_{zy}+\sigma_{xx}\sigma_{yy}\sin^2\psi+\sigma_{xx}(\sigma_{yz}+\sigma_{zy})\sin\psi\cos\psi, \tag{24c}$$

$$\gamma=-\sigma_{zz}\sigma_{xy}\sigma_{yx}+\sigma_{xy}\sigma_{yz}\sigma_{zx}+\sigma_{yx}\sigma_{zy}\sigma_{xz}-\sigma_{yz}\sigma_{zy}\sigma_{xx}-\sigma_{zx}\sigma_{xz}\sigma_{yy}+\sigma_{zz}\sigma_{xx}\sigma_{yy}. \tag{24d}$$

Since we are interested in the high-field limit, we retain only terms of lowest order in $1/H_0$. The elements σ_{ij} are then given by Eqs. (10), (11). Thus,

$$\alpha\cong\sigma_{zz}\cos^2\psi, \tag{25a}$$

$$\beta\cong\sigma_{zz}(\sigma_{xx}\cos^2\psi+\sigma_{yy})+(\sigma_{yx}\sin\psi+\sigma_{zx}\cos\psi)^2+\sigma_{zy}{}^2, \tag{25b}$$

$$\gamma\cong\sigma_{zz}\sigma_{yx}{}^2. \tag{25c}$$

From the evaluation of orders of magnitudes (most conveniently for the case of an ellipsoidal Fermi surface, Sec. V) is may be seen that

$$4\alpha\gamma/\beta^2\gg1, \tag{26}$$

provided that $n_e\tilde{\neq}n_h$. (If $n_e=n_h$ Alfvén waves[1] may propagate.) This enables us to replace Eq. (24) by

$$c^2q^2/2\pi\omega=\pm2(\gamma/\alpha)^{1/2}+i(\beta/\alpha). \tag{27}$$

We shall now simplify the discussion by limiting the frequency of the wave to values much smaller then the collision frequency ($\omega\ll\nu$). This, in fact, is the standard situation encountered in most helicon-wave experiments in metals. The elements of the conductivity tensor then become independent of the frequency and real.

The imaginary term in Eq. (27) is much smaller than the real term, and we may write

$$q^2\cong(\mathrm{Re}q)^2+2i(\mathrm{Re}q)(\mathrm{Im}q). \tag{28}$$

Since only the upper sign in Eq. (27) gives rise to a propagating wave,[26] substitution of Eqs. (25a), (25c),

[26] In semiconductors, the displacement current has to be taken in account; a real positive term has to be added then to the right-hand side of Eq. (27). Thus, when condition (18) does not hold, waves corresponding to the lower sign in Eq. (27) can also propagate.

and (10d) yields

$$(\mathrm{Re}q)^2=\frac{4\pi\omega}{c^2|\cos\psi|}|\sigma_{yx}|=\frac{4\pi|n_e-n_h|e}{cH_0|\cos\psi|}\omega. \quad (29)$$

This is the well-known dispersion relation for helicon waves in the high-field limit. Its most important feature is that, reflecting a similar property of the Hall coefficient, it does not depend on band structure. We may also calculate the refractive index N, and for a single group of charge carriers, using Eqs. (29), (19), and (3) (with the free-electron mass), we get

$$N=c\,\mathrm{Re}q/\omega=\omega_P/(\omega\omega_c|\cos\psi|)^{1/2}. \quad (30)$$

Even for the highest magnetic fields available, N for ordinary metals is much larger than the static refractive index $\sqrt{K}$. This justifies condition (18) for neglecting the displacement current. It is also clear that, for any angle of incidence of the wave, inside the metal it will propagate in a direction practically normal to the surface of incidence.

The imaginary term in Eq. (27), i.e., the damping, will be treated in Sec. IV, and we now turn to a discussion of the polarization of the wave. Solving Eqs. (23b) and (23c) for E_z/E_x and E_y/E_x and keeping only terms of lowest order in $1/H_0$, we obtain with the help of Eq. (29) the result that

$$E_z/E_x=-(\sigma_{yx}\sin\psi+\sigma_{zx}\cos\psi\pm i\sigma_{zy})/\sigma_{zz}\cos\psi, \quad (31)$$

$$E_y/E_x=\pm i/\cos\psi. \quad (32)$$

In Eqs. (31) and (32), the upper sign should be used when $(n_e-n_h)\cos\psi>0$, and the lower sign in the opposite case. This means that the sense of rotation of the electric field corresponds to the sense of rotation of the majority carrier group.

It is not difficult to show that E_z/E_x is of the order of qR when $ql\gg1$, and of the order of ν/ω_c when $ql\ll1$. Thus, we conclude that the electric field is polarized in a plane almost normal to $\mathbf{H}_0$. According to Eq. (32) the polarization is elliptical for $\psi\neq0$ and circular only for $\psi=0$.

The components of $\mathbf{E}$ in the plane normal to $\mathbf{q}$ are E_x and $E_\eta\cong E_y\cos\psi$; therefore, according to Eq. (32) the projection of $\mathbf{E}$ on a plane of constant phase is circularly polarized for arbitrary ψ.

By Maxwell's equations, the magnetic field $\mathbf{H}$ of the wave is always polarized in the plane normal to $\mathbf{q}$. The polarization is close to circular, since from Eq. (16)

$$H_\eta/H_x=-E_x/E_\eta\cong\pm i. \quad (33)$$

It is clear that, to the extent that the small longitudinal component E_z may be neglected, the polarization of the wave does not depend on the shape of the Fermi surface.

IV. DAMPING

We represent the damping by the ratio of the imaginary and real parts of the wave vector.[27] Using Eqs. (28), (27), (25a), (25b), and (29), we find that[28]

$$\Gamma\equiv\frac{\mathrm{Im}q}{\mathrm{Re}q}=\frac{\sigma_{xx}\cos^2\psi+\sigma_{yy}+[(\sigma_{yx}\sin\psi+\sigma_{zx}\cos\psi)^2+\sigma_{zy}{}^2]/\sigma_{zz}}{(4|\sigma_{yx}||\cos\psi|)}. \quad (34)$$

Substitution of the elements σ_{ij} from Eqs. (10) yields a very general, but at the same time very complicated expression. This expression gives the damping for a closed Fermi surface of arbitrary shape, arbitrary value of q_zl, and arbitrary inclination of $\mathbf{H}_0$ and $\mathbf{q}$ with respect to the crystalline axes. It is convenient to break it up into the collisional-damping component $\lim_{q_zl\to0}\Gamma$ and the Landau-damping component $\Gamma-\lim_{q_zl\to0}\Gamma$. We shall understand it better by considering several special cases.

A. $q_zl\ll1$ (Local Limit)

In this limit, with ω neglected, the resonant denominators in the conductivity reduce to ν. The damping becomes independent of q and there is no Landau damping. We remain with the collisional damping, which is given by the expression

$$\Gamma=\frac{\pi c}{eh^3|n_e-n_h|H_0\cos\psi}\Big\{\sum\int_{-p_{zm}}^{p_{zm}}dp_z\,\nu m_c(\langle P_x{}^2\rangle+\langle P_y{}^2\rangle\cos^2\psi)+\Big[\Big(\sum\int_{-p_{zm}}^{p_{zm}}dp_z\,m_c(\langle v_yP_y\rangle\sin\psi+\langle v_zP_y\rangle\cos\psi)\Big)^2+\Big(\sum\int_{-p_{zm}}^{p_{zm}}dp_z\,m_c\langle v_zP_x\rangle\Big)^2\Big]\Big/\Big(\sum\int_{-p_{zm}}^{p_{zm}}dp_z\frac{m_cv_0{}^2}{\nu}\Big)\Big\}. \quad (35)$$

Since the current $\mathbf{J}$ is at all times perpendicular to the wave vector $\mathbf{q}$, when $\mathbf{H}_0\|\mathbf{q}(\psi=0)$ the electrons drift in a

[27] The damping is frequently represented by the ratio of the imaginary and real parts of the frequency, the wave vector being considered a real quantity. For a quadratic dispersion law and small damping, it is easy to show that the "damping" in this representation is simply twice the "damping" in our representation.

[28] The damping may be also expressed in terms of only four elements ($\rho_{\xi\xi},\rho_{\eta\eta},\rho_{\xi\eta},\rho_{\eta\xi}$) of the resistivity tensor, with the ζ axis along q. This was done by F. G. Bass, A. Ya. Blank, and M. I. Kaganov, Zh. Eksperim. i Teor. Fiz. **45**, 1081 (1963) [English transl.: Soviet Phys.—JETP **18**, 747 (1964)] and by P. A. Penz, J. Appl. Phys. **38**, 4047 (1967). Of course, such a representation is relatively simple only as long as the elements $\rho_{\alpha\beta}$ are not expressed in terms of details of the Fermi surface.

plane normal to $\mathbf{H}_0$, at almost a right angle to the electric field. Collisional damping arises owing to a deviation ν/ω_c from the right angle. When $\psi\neq 0$, there is a drift of electrons along the magnetic field as well. Kaner and Skobov[13,14] consider only this component of the current, although the component normal to $\mathbf{H}_0$ is of the same order of magnitude and present even when $\psi=0$.

The dependence of the collisional damping on ψ may be complicated, although it remains of the order of ν/ω_c for any directions of $\mathbf{q}$ and $\mathbf{H}_0$. In the special case when the magnetic field is taken parallel to the wave vector, Eq. (35) simplifies to

$$\Gamma=\frac{\pi c}{eh^3|n_e-n_h|H_0}\nu\left\{\sum\int_{-p_{zm}}^{p_{zm}}dp_z\,m_c\langle P_x^2+P_y^2\rangle+\left[\left(\sum\int_{-p_{zm}}^{p_{zm}}dp_z\,m_c\langle v_zP_y\rangle\right)^2+\left(\sum\int_{-p_{zm}}^{p_{zm}}dp_z\,m_c\langle v_zP_x\rangle\right)^2\right]\Big/\left(\sum\int_{-p_{zm}}^{p_{zm}}dp_z\,m_cv_0^2\right)\right\}. \quad (36)$$

We have assumed here that ν does not depend on p_z, and that it has the same value for all sheets of the Fermi surface. If the shape of the Fermi surface is known, the expression in the curly brackets may be computed for a given direction of $\mathbf{q}$ with respect to the crystalline axes. A measurement of the damping Γ then determines the collision frequency ν. The condition $q_zl\ll 1$ would make such an experiment particularly simple. It would also have the advantage of the absence of electric contacts or high currents which may damage the crystal. The anisotropy or ν may be tested by performing the calculation and the experiment for various directions of $\mathbf{q}$.

For an anisotropic Fermi surface Γ differs from $\nu/2\omega_c$ as was explicitly pointed out by one of the authors[29] in reply to a suggestion by Kao.[30] Obviously, the anisotropy of the Fermi surface affects the collisional damping not only via ν and ω_c. The anisotropy of the collisional damping of helicons was investigated in lead telluride by Schilz.[31]

B. $q_zl\gg 1$ (Extreme Nonlocal Limit)

In this limit, the denominators $(\nu+iq_zv_0)^{-1}$ in the dissipative part of the conductivity may be replaced by $\pi\delta(q_zv_0)$. We obtain the "pure" Landau damping, i.e., unmodified by collisions. The δ function restricts the integration to those electrons whose average velocity v_0 is zero (more accurately ω/q_z, which, in view of our approximation $\omega\ll\nu$ was neglected). Thus, no other electrons participate in the damping process. Due to the centrosymmetry of the Fermi surface, central orbits have $v_0=0$, though for a complex Fermi surface v_0 may happen to vanish for noncentral orbits as well. Obviously, such orbits are closed (there is no steady motion along $\mathbf{H}_0$), and the collisionless damping has practically only a "magnetic" part.[8]

It is not difficult to show that the third term in the square brackets in Eq. (34) is $\sim q_zl$ times smaller than the first two terms. The expression for the damping has been derived already by Kaner and Skobov[13] in their Eq. (198). The damping in our notation[27] is

$$\Gamma=\frac{\pi^2cq}{eh^3|n_e-n_h|H_0}\sum\frac{m_c}{\partial v_0/\partial p_z}\times[(\langle v_yP_y\rangle\sin\psi+\langle v_zP_y\rangle\cos\psi)^2+(\langle v_zP_x\rangle)^2]_{v_0=0}. \quad (37)$$

In general, this expression is of the order of qR. McWhorter and Walpole[15,16] have calculated the power absorbed from the wave in the same limit and their result is equivalent to Eq. (37). However their interpretation of the result seems to us to be oversimplified. They claim that the damping is determined by the average over the cyclotron orbit of the power delivered by the wave to the resonant electrons. Although their result involves the expression $e\mathbf{E}\cdot\mathbf{v}$ (where $\mathbf{E}$ is a complex vector), its imaginary part has no simple physical meaning. This part must be kept, since it appears in a quadratic form.

In the intermediate case, $q_zl\sim 1$, the third term in the curly brackets in Eq. (34) is of the same order (ν/ω_c) as the first two terms. The integration in Eqs. (10) has to be performed over all values of p_z and therefore, electrons with a finite average velocity v_0 also take part in the damping of the wave. Their orbital motion, as before, gives rise to magnetic Landau damping, while their steady motion along the magnetic field gives rise to ordinary Landau damping. Obviously, the damping depends on ν and therefore we speak of "collision-modified" Landau damping. Its mechanism has been discussed in the Introduction (Sec. I).

C. $\psi=0$ ($\mathbf{H}_0\|\mathbf{q}$)

In addition, we assume that $\mathbf{H}_0$ and $\mathbf{q}$ lie in a mirror plane. We may then use Eqs. (15). Substituting these

[29] P. Halevi, Phys. Letters **29A**, 426 (1969).
[30] Y. H. Kao, Phys. Letters, **28A**, 168 (1968).
[31] W. Schilz, Phys. Status Solidi **29**, 559 (1968).

and Eqs. (10c) and (10d) in Eq. (34), we obtain

$$\Gamma=\frac{\pi c}{eh^3|n_e-n_h|H_0}\Big\{\sum\int_{-p_{zm}}^{p_{zm}}dp_z\,m_c\Big[\nu\langle P_x^2+P_y^2\rangle+\frac{(q\langle v_zP_y\rangle)^2}{\nu+iqv_0}\Big]+\Big[\sum\int_{-p_{zm}}^{p_{zm}}dp_z\frac{\nu m_c\langle v_zP_y\rangle}{\nu+iqv_0}\Big]^2\Big/\Big[\sum\int_{-p_{zm}}^{p_{zm}}dp_z\frac{m_cv_0^2}{\nu+iqv_0}\Big]\Big\}. \quad (38)$$

For a complicated Fermi surface the velocity $\mathbf{v}$ of an electron has, in general, an oscillating component along the magnetic field. Then, $\langle v_zP_y\rangle\neq 0$, and we arrive at the conclusion that: For a complex Fermi surface, the Landau damping does not vanish when $\mathbf{H}_0\|\mathbf{q}$. This has been already stated by Quinn[17] and by McWhorter and Walpole.[12,15] The damping was ascribed to the presence of electron orbits tilted with respect to the magnetic field.

McWhorter and Walpole[12] point out that the Landau damping does not vanish even when $\mathbf{H}_0$ and $\mathbf{q}$ are parallel to a symmetry axis of the crystal. This conclusion holds for semimetals, where the symmetry of the Fermi surface is achieved by several ellipsoids tilted at different angles with respect to the symmetry axis. Then there is a positive contribution to the damping from each ellipsoid. The situation is different, however, in the case of ordinary metals. Here, the symmetry is achieved by every sheet of the Fermi surface *separately*. The expression $\langle v_zP_y\rangle$ vanishes for all the cyclotron orbits (even though v_z is not necessarily constant along the orbit). Then by Eq. (38), there will be no collisionless damping, however complicated the Fermi surface of the metal may be.

Whenever there is a finite damping, quantum oscillations of the de Haas–van Alphen type may be expected at temperatures such that $kT\lesssim\hbar\omega_c$. From Eq. (38) it follows that for $\mathbf{H}_0\|\mathbf{q}$ Landau damping may exist due only to tilted cyclotron orbits. Indeed, Miller and Kwok[32] have shown that, in the presence of tilted orbits, oscillations may be expected even when $qR\ll 1$. Such oscillations have been observed by several authors.[19,20,33]

D. Minima in Damping

Let us consider a special cyclotron orbit, along which, at every point, the "orbital" velocity $\mathbf{v}-\mathbf{v}_0$ of the electron is perpendicular to the wave vector $\mathbf{q}$, i.e.,

$$(\mathbf{v}-\mathbf{v}_0)\cdot\mathbf{q}=0. \quad (39)$$

Such an orbit does not contribute to the Landau-damping terms of σ_{ij} [the second terms of Eqs. (10a), (10b), (10e), and (10f)] since by Eq. (39)

$$\mathbf{q}\cdot\langle\mathbf{v}P_{x,y}\rangle=q\cdot v_0\langle P_{x,y}\rangle=0. \quad (40)$$

For an arbitrary direction of the magnetic field with respect to the wave vector, in general, there will be no orbit for which Eq. (39) holds. If, however, for a special orientation of the magnetic field, Eq. (39) holds approximately for a substantial fraction of electron states on the Fermi surface, there will be a minimum in the Landau damping. Such minima were observed in indium by Krylov[19] and by Halevi, Lipson, and Rabinovitch.[20] They are most conveniently detected experimentally when $q_zl\gg 1$, since when this is the case Landau damping dominates over collisional damping, and usually only central orbits (with $v_0=0$) participate in the damping process. For such orbits, Eq. (39) reduces to the simple requirement that the Fermi velocity be perpendicular to the wave vector. With some luck, the angles ψ for which there are minima in the damping may then be deduced from a geometrical construction.[20]

In the case $q_zl\gg 1$, the dynamics of an electron is not modified by collisions and the effect has a simple interpretation. The motion of each electron in real space may be compounded from two parts (Fig. 1). Firstly, it has a constant mean velocity v_0 along the static field $\mathbf{H}_0$. In addition, the electron performs some orbit at frequency ω_c, and has an "orbital" velocity $\mathbf{u}$. Thus,

$$\mathbf{v}=\mathbf{u}+\mathbf{v}_0. \quad (41)$$

Because of the anisotropy of the Fermi surface, the orbit is not necessarily perpendicular to $\mathbf{H}_0$, and, in general, not even plane. If the electron has passed the origin at a time $t=0$, its subsequent position is given by

$$\mathbf{r}=\mathbf{v}_0t+\mathbf{R}, \quad (42)$$

where $\mathbf{R}$ is the instantaneous "radius vector" respective to the "guiding center." Thus at a point $\mathbf{r}$ the electron experiences an electric field[34]

$$\mathbf{E}=\mathbf{E}_0e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}=\mathbf{E}_0e^{i(\mathbf{q}\cdot\mathbf{v}_0-\omega)t+i\mathbf{q}\cdot\mathbf{R}}. \quad (43)$$

Now for $q_zl\gg 1$, the only electrons which can participate in the damping are those who have an average velocity component $\mathbf{v}_0\cdot\mathbf{q}/q$ along the wave vector equal to the phase velocity ω/q. These electrons experience, *on the average*, a constant electric field. In other words, after completion of each cyclotron period the electron "catches up" with the wave. By Eq. (43) there is still a variation along the orbit,

$$\mathbf{E}=\mathbf{E}_0e^{i\mathbf{q}\cdot\mathbf{R}}. \quad (44)$$

The average power absorbed by the electron from the wave during a cyclotron period is, to a very good

[32] P. B. Miller and P. C. Kwok, Phys. Rev. **161**, 629 (1967).
[33] C. C. Grimes, Ref. 1(a), p. 87; J. P. Krylov, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **8**, 1 (1968) [English transl.: Soviet Phys.—JETP Letters **8**, 3 (1968)].

[34] $\mathbf{E}_0$ is a complex vector, independent of $\mathbf{r}$ and $\mathbf{t}$. The real electric field is given by the real part of Eq. (43).

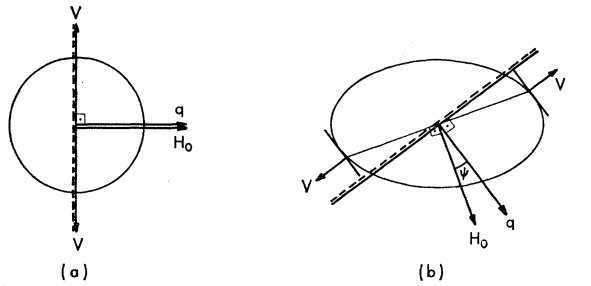

FIG. 2. Illustration of minima in the Landau damping: (a) Isotropic Fermi surface with $\mathbf{H}_0\|\mathbf{q}$; (b) Anisotropic Fermi surface with $\mathbf{H}_0$ tipped with respect to $\mathbf{q}$ at an angle determined by the condition $\mathbf{v}\perp\mathbf{q}$. In both cases the cyclotron orbit in real space (solid line) lies in a plane of constant phase (dashed line). The electron experiences a constant electric field from which, in the average, it cannot draw energy. (Time variation of $\mathbf{E}$ is neglected.)

approximation[35]

$$P=e\langle\mathbf{E}\cdot\mathbf{u}\rangle=e\mathbf{E}_0\cdot\langle\mathbf{u}\exp(i\mathbf{q}\cdot\mathbf{R})\rangle. \tag{45}$$

Although we assumed $qR\ll1$, the electric field may not be taken as constant. By Eq. (41), $\langle\mathbf{u}\rangle=0$, and such an approximation would lead to a vanishing power. *Thus, Landau damping arises precisely because of the small variation of the field along the cyclotron orbit,* given by Eq. (44).

Now consider a special orbit, along which, at every point, $\mathbf{R}\perp\mathbf{q}$. In other words, the magnetic field $\mathbf{H}_0$ is tipped at such an angle ψ with respect to $\mathbf{q}$ that the normal to the orbit (the vector $\mathbf{A}$ in Fig. 1) is parallel to $\mathbf{q}$. This, of course, can be realized only for a plane orbit, and is equivalent to Eq. (39). In this case we get from Eq. (45)

$$P=e\mathbf{E}_0\cdot\langle\mathbf{u}\rangle=0. \tag{46}$$

In summary, for such orientations of the magnetic field that the plane of the orbit is normal to the wave vector, the electron experiences a constant electric field. It is speeded up by this field during one-half of the cyclotron period and slowed down during the other half in such a way that the energy interchange with the wave comes to zero. Figures 2 and 3 illustrate the effect in a simple manner by comparison to the isotropic case.

Of course, realistic electron orbits are not plane, and even in the case $q_zl\gg1$, Eq. (39) will not hold exactly. Therefore, only minima and not actual zeros in the damping are to be expected. For a given orientation of $\mathbf{q}$ with respect to the crystalline axes there may be several minima in the damping, and none of them at $\psi=0$. (This, indeed, is the situation in the experiments on indium.[20]) On the other hand, in an isotropic electron plasma there is only a single zero in the Landau damping at $\psi=0$. In this respect, the damping of helicon waves in a real metal differs *qualitatively* from that in a free-electron plasma.

In simple cases measurement of the angle ψ for minimum damping may provide information about the Fermi surface.

V. ELLIPSOIDAL FERMI SURFACE

The equation of an ellipsoidal Fermi surface in a coordinate system coinciding with the principal axes is

$$\epsilon=\frac{p_x^2}{2m_1}+\frac{p_y^2}{2m_2}+\frac{p_z^2}{2m_3}. \tag{47}$$

We restrict $\mathbf{q}$ and $\mathbf{H}_0$ so as to have no x component, i.e., to a mirror plane of the ellipsoid. Their orientations with respect to the p_y axis are ϕ and θ, respectively (Fig. 4). Since $\mathbf{q}$ is perpendicular to the surface of incidence of the wave, the tipping angle is $\psi=\theta-\phi$. We now transform to a new coordinate system by rotating around the p_x axis through an angle θ. In this system $\mathbf{H}_0$ points in the z direction and the components of $\mathbf{q}$ are given by Eqs. (9), as required by our conventions in Sec. II. The equation of the surface in the new system reads

$$\epsilon=Ap_x^2+Bp_y^2+Cp_z^2+Dp_yp_z, \tag{48a}$$

where

$$A=1/2m_1, \tag{48b}$$

$$B=(\sin^2\theta/2m_2)+(\cos^2\theta/2m_3), \tag{48c}$$

$$C=(\cos^2\theta/2m_2)+(\sin^2\theta/2m_3), \tag{48d}$$

$$D=[(1/m_2)-(1/m_3)]\sin\theta\cos\theta. \tag{48e}$$

The cyclotron orbits in momentum space are ellipses of area

$$S=\pi(AB)^{-1/2}[\epsilon-(C-D^2/4B)p_z^2], \tag{49}$$

as may be easily derived from Eqs. (48). According to the definition given in Eq. (2), the cyclotron mass is

$$m_c=(4AB)^{-1/2}=[m_1m_2m_3/(m_2\cos^2\theta+m_3\sin^2\theta)]^{1/2}. \tag{50}$$

Thus, m_c is independent of p_z, and changes monotonically from $(m_1m_3)^{1/2}$ to $(m_1m_2)^{1/2}$ as θ is increased from 0° to 90°.

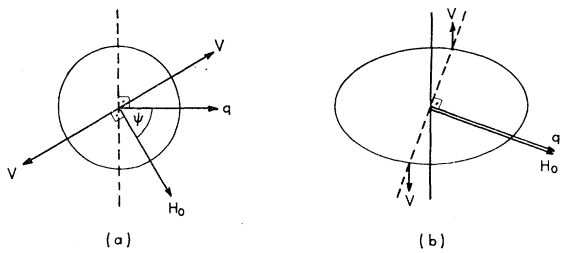

FIG. 3. Landau damping arises when the cyclotron orbit in real space (solid line) moves our of a plane of constant phase (dashed line). For an isotropic Fermi surface this happens when $\mathbf{H}_0$ is tipped at an angle with respect to $\mathbf{q}$, while for an anisotropic Fermi surface it may happen, in general, even when $\mathbf{H}_0\|\mathbf{q}$.

[35] In. Eq. (45) we neglect a term $e\langle E_z\rangle v_0$, which represents the ordinary "electric" Landau damping (Ref. 8). This is legitimate since $v_0=\omega/q_z\ll u\sim v_F$ and also $E_z/E_x\sim qR\ll1$. Moreover, it is not difficult to see from Eqs. (31) and (10d)–(10f) that when Eq. (39) holds for all electron orbits, $E_z=0$.

We shall now solve the equation of motion of an electron. Using Eqs. (5a), (4), (48a), and (50), we obtain

$$dp_x/d\tau = -(B/A)^{1/2}p_y - [D/2(AB)^{1/2}]p_z. \quad (51)$$

If we assume a solution of the form $K\cos\tau$ for p_x, Eq. (51) may be solved for p_y. The coefficient K is found by substituting p_x and p_y in Eq. (48a). Using Eq. (49), the results may be written in the form

$$p_x = [\pi^{-1}(B/A)^{1/2}S]^{1/2}\cos\tau, \quad (52a)$$

$$p_y = [\pi^{-1}(A/B)^{1/2}S]^{1/2}\sin\tau - (D/2B)p_z. \quad (52b)$$

For the velocity components, we get from Eqs. (4) and (52)

$$v_x = 2Ap_x = 2A[\pi^{-1}(B/A)^{1/2}S]^{1/2}\cos\tau, \quad (53a)$$

$$v_y = 2Bp_y + Dp_z = 2B[\pi^{-1}(A/B)^{1/2}S]^{1/2}\sin\tau, \quad (53b)$$

$$v_z = 2Cp_z + Dp_y = D[\pi^{-1}(A/B)^{1/2}S]^{1/2}\sin\tau + (2C - D^2/2B)p_z. \quad (53c)$$

The x and y components of the velocity have only an oscillating part, hence the average velocity is along $\mathbf{H}_0$ (as it should be in general). Its magnitude is

$$v_0 = (2C - D^2/2B)p_z = (m_c^2/m_1m_2m_3)p_z. \quad (54)$$

If $D\neq 0$, the orbital velocity $\mathbf{v}-\mathbf{v}_0$ has a component in the z direction [the first term of Eq. (53c)], reflecting the fact that the orbit in real space is not normal to $\mathbf{H}_0$. If $m_2\neq m_3$, D may vanish only when $\mathbf{H}_0$ is along one of the directions of high symmetry ($\theta=0°$, 90°, or 180°).

The "mean free path" l will be defined as the average distance traversed between collisions along the magnetic field by an electron with maximal average velocity $\mathbf{v}_0$:

$$l = v_0(p_{zm})/\nu = m_c^2 p_{zm}/m_1m_2m_3\nu. \quad (55)$$

The calculation of the conductivity tensor is given in the Appendix. It is assumed that $\omega \ll \nu$ and that ν does not depend on p_z. Substituting Eqs. (A9a)–(A9f) into Eq. (34), with some algebraic manipulations, we obtain the damping[36]:

$$\Gamma = \frac{(m_3/m_1)^{1/2}}{2|\cos\psi|}\frac{\nu}{\omega_0}\left[\frac{1}{2}\left(\frac{m_1}{m_3}+\frac{m_2}{m_3}\sin^2\phi+\cos^2\phi\right) + \chi(q_zl)\frac{\cos^2\phi}{\tan^2\theta + m_2/m_3}\times\left(\tan\theta - \frac{m_2}{m_3}\tan\phi\right)^2\right], \quad (56a)$$

$$\chi(q_zl) = \frac{3}{8}\frac{(q_zl)^2\tan^{-1}(q_zl)}{q_zl - \tan^{-1}(q_zl)} - \frac{9}{8}, \quad (56b)$$

$$\omega_0 = eH_0/(m_1m_3)^{1/2}c. \quad (56c)$$

[36] In Ref. 21, a certain term in the damping was erroneously classified as belonging to the collisional damping, whereas, in fact, it belongs to the collision-modified Landau damping. Because of this error, the collisional damping was not given correctly there.

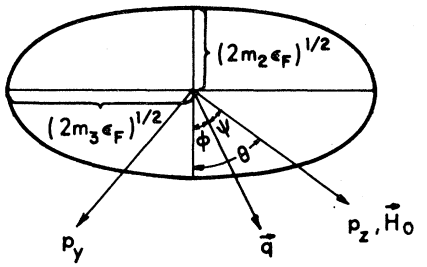

FIG. 4. Cross section $p_x=0$ of an ellipsoidal Fermi surface illustrating the orientations of $\mathbf{H}_0$ and $\mathbf{q}$ with respect to the principal axes.

The last expression represents the cyclotron frequency for $\theta=0$. We shall now examine Eq. (56a) for several special cases.

A. $m_1=m_2=m_3=m$ (Isotropic Fermi Surface)

In this case, Eq. (56a) reduces to

$$\Gamma = (1/2|\cos\psi|)(\nu/\omega_0)[1+\chi(q_zl)\sin^2\psi]. \quad (57)$$

This equation checks with the result of Buchsbaum and Platzman,[8] the function χ being given now in a compact form. The Landau damping is given by the second term in Eq. (57). It vanishes for $\psi=0$, and is an increasing function of $|\psi|$.

B. $q_zl \ll 1$ (Local Limit)

The function χ [Eq. (56b)] may be expanded in a power series of q_zl as follows:

$$\chi(q_zl) = \tfrac{3}{10}(q_zl)^2 - \frac{27}{350}(q_zl)^4 + \cdots. \quad (58)$$

Thus, $\chi\to 0$ in the limit $q_zl\to 0$, and the second term in Eq. (56a) vanishes. The damping is given by

$$\Gamma = \frac{(m_3/m_1)^{1/2}}{4|\cos\psi|}\frac{\nu}{\omega_0}\left(\frac{m_1}{m_3}+\frac{m_2}{m_3}\sin^2\phi+\cos^2\phi\right). \quad (59)$$

This expression represents the collisional damping, since there is no Landau damping in the limit $q_zl\to 0$. The second term in Eq. (56a) then represents the Landau damping, modified by collisions.[36]

We note that, even in this relatively simple case, the damping does not reduce to Kao's expression $\nu/2\omega_c$.[29,30] The dependence on the tipping angle is simply $1/\cos\psi$.

C. $q_zl \gg 1$ (Extreme Nonlocal Limit)

In this case, we expand χ in power series of $(q_zl)^{-1}$.

$$\chi(q_zl) = \tfrac{3}{16}\pi\, q_zl - \tfrac{1}{32}(48-3\pi^2) + \tfrac{1}{64}(3\pi^3-24\pi)(q_zl)^{-1} + \cdots. \quad (60)$$

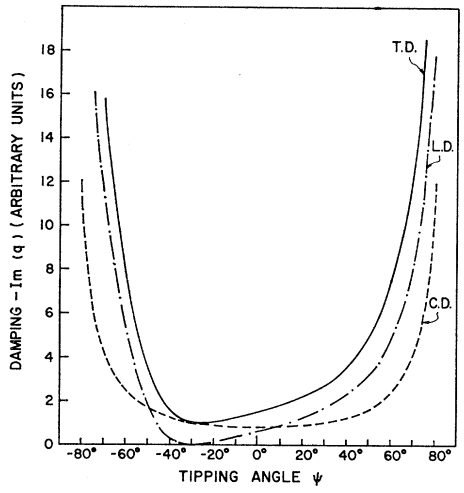

FIG. 5. Collisional (CD), Landau (LD), and total (TD) dampings for the case $m_1=m_2=\frac{1}{3}m_3$ and $\phi=60°$. According to Eq. (63), the Landau damping vanishes for $\theta=30°$, i.e., $\psi=-30°$. (a) $ql=10$, $\omega_0/\nu=100$; (b) $ql=50$, $\omega_0/\nu=500$.

Substituting the first term in the expansion in Eq. (56a) and neglecting collisional damping, we obtain

$$\Gamma=\frac{3\pi}{32}\left(\frac{m_3}{m_1}\right)^{1/2}\frac{\cos^2\phi}{\tan^2\theta+m_2/m_3}\times\left(\tan\theta-\frac{m_2}{m_3}\tan\phi\right)^2\frac{q_z v_0(p_{zm})}{\omega_0}. \quad (61)$$

This is the genuine collisionless damping; we see that, in general, it is of the order qR.

The second term in the expansion (60) gives a contribution to the damping which is of the same order of magnitude as the collisional damping. It represents the lowest-order correction to the collisionless damping due to collisions.

D. $\psi=0$ ($\mathbf{H}_0\|\mathbf{q}$)

We substitute $\psi=0$ and $\theta=\phi$ in Eq. (56a) and get

$$\Gamma=\frac{1}{2}\left(\frac{m_3}{m_1}\right)^{1/2}\frac{\nu}{\omega_0}\left\{\frac{1}{2}\left(\frac{m_1}{m_3}+\frac{m_2}{m_3}\sin^2\phi+\cos^2\phi\right)+\chi(ql)\left(1-\frac{m_2}{m_3}\right)^2\frac{\sin^2\phi}{\tan^2\phi+m_2/m_3}\right\}. \quad (62)$$

Thus, for $m_2\neq m_3$, the Landau damping vanishes only when $\mathbf{q}$ and $\mathbf{H}_0$ are along one of the principal axes of the ellipsoid.[12,15,17,21]

E. Mimina in Damping

A glance at Eq. (56a) reveals that, when

$$\tan\theta=(m_2/m_3)\tan\phi, \quad (63)$$

the Landau damping vanishes and we remain with the collisional damping.[21] This result holds for arbitrary $q_z l$. Since, for a finite $q_z l$, all cyclotron orbits contribute to the damping, we may guess that when the orientation of $\mathbf{H}_0$ is given by Eq. (63), then $(\mathbf{v}-\mathbf{v}_0)\cdot\mathbf{q}=0$ [Eq. (39)] must hold for *all* cyclotron orbits. The truth of this assertion is evident from Eq. (A3).

The clear-cut situation is due to a special geometric property of the ellipsoidal Fermi surface. Because of this property, all orbits in real space are planar, parallel to each other, and normal to the vector $\tilde{\mathbf{m}}\cdot\mathbf{H}_0$, where $\tilde{\mathbf{m}}$ is the mass tensor.[15] It is easy to show that this vector is inclined with respect to the p_y axis (in the principal axes coordinate system) at an angle given by

$$\tan\alpha=(m_3/m_2)\tan\theta. \quad (64)$$

When the plane of the orbit is perpendicular to the wavevector, i.e., when $\alpha=\phi$, the inclincation of $\mathbf{H}_0$ with respect to $\mathbf{q}$ is indeed given by Eq. (63).

Thus, at least in the somewhat unrealistic case of a single-ellipsoidal Fermi surface, the Landau damping may vanish, even when $\mathbf{H}_0$ is not parallel to a symmetry axis of high order (in disagreement with a statement in Refs. 13 and 14).

A simple geometrical construction, relating $\mathbf{H}_0$ and $\mathbf{q}$ at zero Landau damping, is given in Ref. 20, Fig. 4.

In Figs. 5(a) and 5(b), we plot the collisional, Landau, and total dampings for two representative cases. A comparison of Fig. 5(a) with Fig. 5(b) shows that, the higher the value of ql, the sharper the minimum in the damping will be. At high values of ql, the total-damping curve practically follows the Landau-damping curve. The effect of the collisional damping is then only to

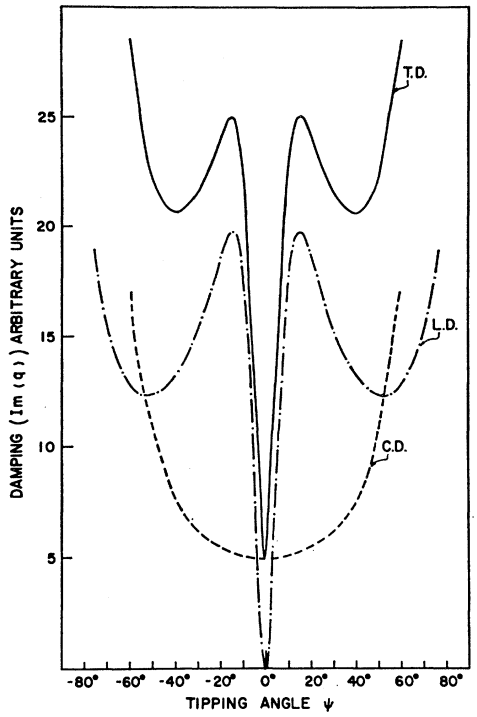

FIG. 6. Collisional (CD), Landau (LD), and total (TD) dampings for the case $m_1=m_2=m_3/30$, $\phi=0°$, $ql=10$, $\omega_0/\nu=100$. The additional structure is associated with a variation of the cyclotron mass.

"lift" the Landau damping by an amount equal to the collisional damping for θ given by Eq. (63).

For sufficiently high values of the anisotropy ratio m_3/m_2, the Landau damping may have other minima (and maxima) as well, but not zeros. These are associated with the variation of the cyclotron mass, Eq. (50), with the direction of $\mathbf{H}_0$. Such minima are shown in Fig. 6, for $\mathbf{q}$ along one of the principal axes of the ellipsoid.

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APPENDIX: CONDUCTIVITY TENSOR FOR AN ELLIPSOIDAL FERMI SURFACE

Since $\mathbf{q}$ and $\mathbf{H}_0$ are chosen to lie in a mirror plane of the ellipsoid, we may use Eq. (14). The averages of other quantities that appear in the conductivity tensor, Eqs. (10), are calculated with the help of Eqs. (48), (50), (52), (53):

$$\langle P_x^2\rangle=(1/2\pi)(m_1/m_c)S, \tag{A1}$$

$$\langle P_y^2\rangle=(1/2\pi)(m_c/m_1)S, \tag{A2}$$

$$\mathbf{q}\cdot\langle \mathbf{v}P_y\rangle=(1/2\pi)q\cos\phi\cos\theta \times[\tan\theta-(m_2/m_3)\tan\phi](m_c/m_1m_2)S. \tag{A3}$$

With these substitutions, and neglecting ω, we obtain

$$\sigma_{xx}=\frac{2c^2\nu m_c^2}{h^3H_0^2m_1}\int_{-p_{zm}}^{p_{zm}}S\,dp_z-\frac{i\pi c^2qm_c^7\cos^2\phi\cos^2\theta}{h^3H_0^2m_1^3m_2^3m_3\cos\psi} \times\left(\tan\theta-\frac{m_2}{m_3}\tan\phi\right)^2\int_{-p_{zm}}^{p_{zm}}\frac{(p_z^2-a)^2}{p_z-b}dp_z, \tag{A4a}$$

$$\sigma_{yy}=\frac{2c^2\nu m_1}{h^3H_0^2}\int_{-p_{zm}}^{p_{zm}}S\,dp_z, \tag{A4b}$$

$$\sigma_{zz}=-\frac{i4\pi e^2m_c^3}{h^3qm_1m_2m_3\cos\psi}\int_{-p_{zm}}^{p_{zm}}\frac{p_z^2\,dp_z}{p_z-b}, \tag{A4c}$$

$$\sigma_{zx}=-\frac{nec}{H_0}\tan\psi+\frac{i2\pi ec\nu m_c^3\cos\phi\cos\theta}{h^3H_0qm_1m_2\cos^2\psi} \times\left(\tan\theta-\frac{m_2}{m_3}\tan\phi\right)\int_{-p_{zm}}^{p_{zm}}\frac{p_z^2-a}{p_z-b}dp_z, \tag{A4e}$$

$$a=2m_1m_2m_3\epsilon_F/m_c^2, \tag{A5}$$

$$b=i\nu m_1m_2m_3/qm_c^2\cos\psi. \tag{A6}$$

The limits of the integrals are obtained from the condition that the plane $p_z=p_{zm}$ is tangent to the ellipsoid. Obviously, we must substitute in the equation of the ellipsoid, Eq. (48), $p_x=0$ and $p_z=p_{zm}$, and equate to zero the determinant of the resulting quadratic equation. We get

$$p_{zm}=(2\epsilon_Fm_1m_2m_3)^{1/2}/m_c. \tag{A7}$$

The integral in Eq. (A4b) is simply the volume of the Fermi surface, which is related to the electron density by Eq. (13):

$$\int_{-p_{zm}}^{p_{zm}}S\,dp_z=\tfrac{4}{3}\pi(2m_1\epsilon_F)^{1/2}(2m_2\epsilon_F)^{1/2}(2m_3\epsilon_F)^{1/2} =\tfrac{1}{2}h^3n. \tag{A8}$$

The other integrands are rational functions and may be decomposed to the sum of a polynomial and a term of the form const/(p_z-b). The integrations are straightforward, and after some algebraic manipulations, we

obtain the following result:

$$\sigma_{xx}=\frac{nec}{H_0}\frac{(m_c^2/m_1)cv}{eH_0}\times\left\{1+\frac{3}{4}\frac{m_3}{m_2}\frac{\cos^2\phi\cos^2\theta}{\cos^2\psi}\left(\tan\theta-\frac{m_2}{m_3}\tan\phi\right)^2\times\left[\frac{1}{r}(1+r^2)^2\tan^{-1}\left(\frac{1}{r}\right)-r^2-\frac{5}{3}\right]\right\},\quad\text{(A9a)}$$

$$\sigma_{yy}=(nec/H_0)m_1cv/eH_0,\quad\text{(A9b)}$$

$$\sigma_{zz}=3\frac{nec}{H_0}\frac{eH_0}{(m_1m_2m_3/m_c^2)cv}r^2\left[1-r\tan^{-1}\left(\frac{1}{r}\right)\right],\quad\text{(A9c)}$$

$$\sigma_{yx}=nec/H_0,\quad\text{(A9d)}$$

$$\sigma_{zx}=-\frac{nec}{H_0}\times\left\{\tan\psi+\frac{3}{2}\frac{m_c^2}{m_1m_2}\frac{\cos\phi\cos\theta}{\cos\psi}\left(\tan\theta-\frac{m_2}{m_3}\tan\phi\right)\times\left[r^2-r(1+r^2)\tan^{-1}\left(\frac{1}{r}\right)\right]\right\},\quad\text{(A9e)}$$

$$\sigma_{zy}=0,\quad\text{(A9f)}$$

$$1/r=p_{zm}/|b|=q_zl,\quad\text{(A10)}$$

where l is given by Eqs. (55) and (A7).

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Low-Temperature Specific-Heat Study of Cu-Pd Alloys

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Variations with composition of the electronic specific-heat coefficient γ and Debye temperature Θ_D have been determined for Cu-Pd alloys from low-temperature specific-heat measurements between 1.5 and 4.2°K. The dependence of γ on composition is similar to that found by Hoare *et al.* for Ag-Pd alloys. For Cu alloys with small concentrations of Pd, deviations from a rigid-band model have been observed. Thermodynamic properties of these alloys are discussed in terms of the band structure deduced from γ values. Changes in long-range order and short-range order have pronounced effects on γ and Θ_D values.

I. INTRODUCTION

THE phase diagram for Cu-Pd shows that there exists a continuous solid solution (fcc) above 600°C, and that below 600°C there are three superlattice structures: α' (ordered fcc), α'' (ordered fct), and β (ordered bcc).[1] In Pd-rich alloys, x-ray diffuse scattering results by Chen[2] indicate local clustering. In this paper we have studied the effects of long-range order in the β phase and local clustering in Pd-rich alloys on the low-temperature specific heats in relation to the energy band structure and the thermodynamic properties. We have also investigated the effect of dilute concentrations of Pd on the electronic specific heat in the Cu-rich alloys.

The effect of superlattice formation (long-range order) on the electronic structure has been considered by several authors.[3–5] Nicholas[5] has suggested that when an alloy develops a superlattice the interaction between the Fermi surface and new Brillouin zone boundaries, corresponding to the extra Bragg reflections, splits the energy band. The density of states at the Fermi surface may change depending upon the position of the Fermi level in the energy band. Furthermore, changes may occur in the lattice contribution to the specific heat.[6]

Chen[2] has observed that the local order coefficients α_1 of Pd-rich alloys are positive indicating clustering and that the degree of clustering increases with temperature. These results were partially substantiated by Myles and Darby[7] using vapor-pressure measurements. These results are somewhat puzzling in view of the phase relationships found in this system. Since the β phase has a superlattice structure (CsCl-type structure) we expect short-range order rather than clustering to prevail at compositions close to the β phase. Furthermore, large negative values of the heat of mixing[8] also suggest that pairs of unlike atoms are formed if a simple

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